

Test 2

① Find the Arc Length from $t=0$ to $t=5$ of the curve $r(t)$ given by

$$r(t) = \langle 5t, \cos(t), \sin(t) \rangle.$$

② Draw Level Curve for the Surface defined by:

$z = f(x, y) = \sqrt{1 - x^2 - y^2}$. In particular, draw the Level curves for $c=0, c=1, c=2$

③ What is the unit Tangent vector of $r(t) = t^2 + 1$ at $t=2$? How about the unit Normal vector of $r(t) = t^2 + 1$ at $t=2$?

(4) Find the Limit.

$$\lim_{(x,y) \rightarrow (0,0)} [e^{(x+1)y} - \sin(x-y+2)]$$

(5) If w is a differentiable function of x and y and $x = h(s, t)$ and $y = g(s, t)$,

and $\frac{\partial x}{\partial s}$, $\frac{\partial x}{\partial t}$, $\frac{\partial y}{\partial s}$, and $\frac{\partial y}{\partial t}$

all exist, and at $s=0$, $t=0$

$x=2$, $y=3$ and

$w = x + y$, and

$$\frac{\partial x}{\partial s}(0) = 1, \quad \frac{\partial y}{\partial s}(0) = 2,$$

$$\frac{\partial x}{\partial t}(0) = 2, \quad \frac{\partial y}{\partial t}(0) = 3,$$

then find $\frac{\partial w}{\partial t}(0)$ and $\frac{\partial w}{\partial s}(0)$.